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Analysis of forced vibrations of conveyor belts

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Abstract. The paper presents the study of forced transverse vibrations of conveyor belts. Those are caused by rollers having an eccentricity, which originates either from production errors or transported material sticking to bearing rollers of the lower conveyor branch. The dynamic behavior of the belt was analyzed by solving a boundary problem described by differential equations expressed in partial derivatives and corresponding boundary conditions. The solution helped to determine modal parameters of the system – eigenfrequencies and eigenforms of vibrations, which, in their main modes, would be the basis of practical recommendations for selecting such parameters of conveyor systems and their operational modes which could prevent the system from getting into a resonance state.

1. Introduction

One advanced international tendency in designing belt conveyors has lately become the practice of raising their performance by substantially increasing the belt velocity without making any significant changes in the overall design.

There are many problems which should be solved both while designing and using the systems of high-speed belt conveyors: starting-breaking modes, loading and unloading of transported material, minimizing dynamic loads on bearing-based assemblies of rollers and conveyor belts, maintaining stability of belt motion, excluding resonance and near-resonance modes, preventing vibrational dynamic behavior of a mechanical system.

Friction and nonlinear rigidity could lead to instability of a mechanical system affected by small or unpredicted levels of forced excitation. Nowadays, the dynamic analysis is often limited to studying such excitations which contain one periodic function or based on various linear models providing a convenient solution. Meanwhile, many dynamic processes in belt conveyors could not be explained via the linear dynamics only.

There are many papers which have been published in recent years – for example, nonlinear forced vibrations are analyzed in [1-3]. The results of experiments which determine dynamic characteristics of belt conveyors are cited in [4, 5].

A promising approach to studying nonlinear processes of chaos in belt conveyor systems by means of a strange attractor analysis is given in [6, 7]. The theory of chaos could help to analyze the stability of belt motion even in those cases when traditional methods fail.

2. Solving the research problem

Consider an equation for transverse vibrations of conveyor belt without factoring in the dissipation of energy:



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$$\frac{\partial^2 w}{\partial t^2} = \alpha \frac{\partial^2 w}{\partial x^2} - \beta \frac{\partial^4 w}{\partial x^4} + g, \quad (1)$$

$$(0 \leq x \leq l; 0 \leq t < \infty)$$

where $\alpha = \frac{Sg}{q}$; $\beta = \frac{D_x g}{q}$; $w(x, t)$ is a belt deflection at an x cross-section; S, D_x are a belt tension and flexural rigidity, correspondingly; $q(x, t)$ is a linear load on the belt from its own and transported material weight (further considerations assume $q(x, t) = q = \text{const}$); $g = 9,81 \text{ m/s}^2$; t is a time.

The boundary conditions could be expressed in the form of kinematic excitation of belt ends resting on bearing rollers whose imbalance is the main cause of the problem:

$$w(0, t) = \delta e^{i\omega t}; w(l, t) = \delta e^{i\omega t}. \quad (2)$$

where ω, δ are a frequency and amplitude of vibrations at the left and right belt ends, correspondingly.

A solution to (1) would be sought after in the form of

$$w(x, t) = w(x) + \frac{l-x}{l} \delta e^{i\omega t} + \frac{x}{l} \delta e^{i\omega t} + y(x, t), \quad (3)$$

where $w(x)$ is a stationary form of belt deflection; $y(x)$ is a new unknown function which satisfies the boundary problem:

$$\frac{\partial^2 y}{\partial t^2} = \alpha \frac{\partial^2 y}{\partial x^2} - \beta \frac{\partial^4 y}{\partial x^4} + f(x, t). \quad (4)$$

Contrary to previous studies [8], where tensile stresses in a belt were thought to be constant in the case of small transverse vibrations, changes in belt tension over time are taken into account as a function of its deflection for a special case when equation (4) is transformed into the form of

$$\frac{\partial^2 y}{\partial t^2} = (\alpha + ay) \frac{\partial^2 y}{\partial x^2} - \beta \frac{\partial^4 y}{\partial x^4} + f(x, t). \quad (5)$$

Now, let us introduce a new unknown function of $\xi = \alpha + ay$ and rewrite (5) as

$$\frac{\partial^2 \xi}{\partial t^2} = \xi \frac{\partial^2 \xi}{\partial x^2} - \beta \frac{\partial^4 \xi}{\partial x^4} + af(x, t). \quad (6)$$

A solution to (6) would have the form of

$$\xi(x, t) = p(t) \cdot \varphi_1(x) \quad (7)$$

where $\varphi_1(x)$ is the first mode of vibrations found by solving the boundary problem.

Substituting (7) into (6), we get:

$$\frac{\partial^2 p}{\partial t^2} \cdot \varphi_1 = p \varphi_1 p \frac{\partial^2 \varphi_1}{\partial x^2} - \beta p \frac{\partial^4 \varphi_1}{\partial x^4} + af(x, t) \quad (8)$$

Let us multiply (8) and φ_1 and x -integrate the product within the distance between two rollers:

$$\frac{\partial^2 p}{\partial t^2} \int_0^l \varphi_1^2 dx = p^2 \int_0^l \varphi_1^2 \frac{\partial^2 \varphi_1}{\partial x^2} dx - \beta p \int_0^l \varphi_1 \frac{\partial^4 \varphi_1}{\partial x^4} dx + af(x, t) \int_0^l \varphi_1 dx \quad (9)$$

Consider a spectral problem, that is a problem concerning eigenforms and eigenvalues of vibrations:

$$\frac{\partial^4 \varphi_1}{\partial x^4} = \lambda_1^4 \varphi_1, \quad (0 \leq x \leq l); \quad (10)$$

for

$$\varphi_1(0) = \varphi_1(l) = 0; \quad \frac{\partial \varphi_1(0)}{\partial x} = \frac{\partial \varphi_1(l)}{\partial x} = 0.$$

Equation (10) is solvable for countable numbers which would be positive roots of the following equation:

$$sh(\lambda_1, l) \cdot \cos(\lambda_1, l) = 1, \quad (11)$$

Derived from which is $\lambda_1 = 4,73/l$.

Eigen forms own such properties as

$$\int_0^l \varphi_1^2(x) dx = 1, \quad (12)$$

and also

$$\int_0^l \varphi_1^2(x) \frac{\partial^2 \varphi_1(x)}{\partial x^2} dx = x_{11} < 0. \quad (13)$$

Transformations give us the Cauchy problem:

$$\frac{\partial^2 p}{\partial t^2} = p^2 x_{11} - \beta \lambda_1^4 p + \alpha f(x, t) \int_0^l \varphi_1 dx, \quad (14)$$

where $f(x, t) = \delta w^2 \sin \omega t$.

Assuming that an $y(t)$ movement is sinusoidal and occurs with the frequency of kinematic excitation ω on rollers, we can express the solution in the following form:

$$p = p_0 + A \sin \omega t. \quad (15)$$

Let us substitute it into (14):

$$\begin{aligned} -\omega^2 \cdot A \sin \omega t = & (p_0 + A \sin \omega t)^2 x_{11} - \beta_1 (p_0 + A \sin \omega t) + \\ & + a \delta \omega^2 \left(\int_0^l \varphi_1(x) dx \right) \sin \omega t, \end{aligned} \quad (16)$$

where $p_0 = \frac{\alpha}{\varphi_1(l/2)}$; $\beta_1 = \beta \cdot \lambda_1^4$.

The maximal (amplitude) movements of the belt caused by vibrations correspond to $\sin \omega t = 1$. Then (16) can be rewritten as

$$\tilde{x}_{11} \cdot B^2 + \beta_1 B = (M - p_0 + B) \omega^2, \quad (17)$$

where the following expressions are assumed:

$$B = p_0 + A; \quad -x_{11} = \tilde{x}_{11}; \quad M = a\delta \int_0^l \varphi_1(x) dx.$$

3. The solution

Referring to [9], we can present a graphical solution to (17) (figure 1).

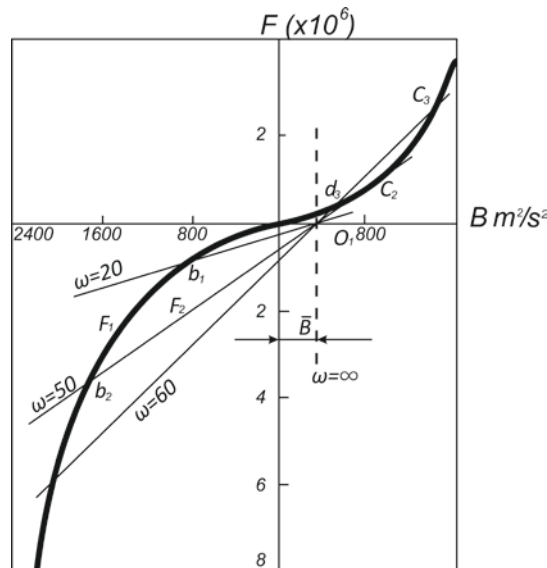


Figure 1. Graphical representation of equation (17).

The X -axis corresponds to B , which is measured in squared velocity (m^2/s^2). The Y -axis represents the functions in the left and right parts of (17):

$$6F_1 = \tilde{x}_{11}B^2 + \beta_1B \text{ and } F_2 = (M - p_0 + B)\omega^2$$

Having the graph of F_1 plotted for $B \geq 0$, we reflect it symmetrically relative to the coordinate origin. The graphs of F_2 are represented by straight lines for fixed ω ($0 \leq \omega < \infty$), all of them crossing the X -axis at one point of $B = p_0 - M$.

The functions F_1 and F_2 cross each other when an instantaneous equilibrium of the system in the end of a peak is achieved. Their X -coordinates allow to find amplitude values of the belt movement from vibrations. Values of the belt deflection in the middle between two rollers correspond to a coefficient of the first mode form $\varphi_1(l/2)$.

Using an "amplification coefficient" term, we can write:

$$A/\delta = \frac{b(\omega) \cdot \varphi_1(l/2)}{a \cdot \delta}. \quad (18)$$

Note that in the region of smaller frequencies of F_1 and F_2 there is only one cross point for every frequency. When the frequency of excitation increases, the straight lines of F_2 are counterclockwise rotated about the point O_1 until the graphs of F_1 and F_2 touch at c_2 . Further increase of the frequency leads to equation (17) having three roots or cross points.

X-coordinates of F_1 and F_2 cross points allow to determine the relation $B(\bar{\omega})$, and then, with the help of (18), find a desired amplitude-frequency characteristic (figure 2). Similarly-designated points in Fig. 2 and Fig. 1 correspond to each other.

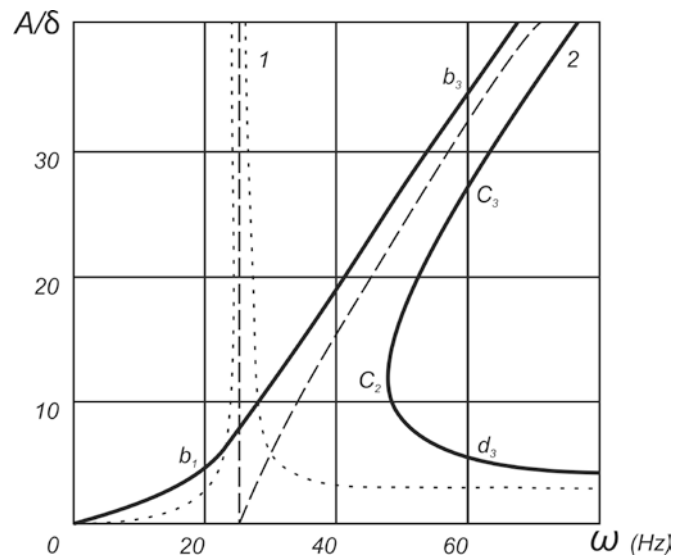


Figure 2. Resonance diagram (amplitude-frequency characteristic) in the case of a variable tension of a conveyor belt under forced transverse vibrations (1 – linear, 2 – nonlinear).

Visually determining the amplitude values of vibrations is quite illustrative, but let us also write a closed-type expression, which originates immediately from the solution to (17).

Considering the fact that the second summand of the function has little influence, an approximate solution could be derived from the following equation:

$$\tilde{x}_{11} \cdot B^2 - \omega^2 B + \omega^2 (p_0 - M) = 0 \quad (19)$$

$$B = \frac{\omega^2}{2\tilde{x}_{11}} \left[1 \pm \sqrt{1 - \frac{4\tilde{x}_{11}}{\omega^2} (p_0 - M)} \right], \quad (20)$$

Now, using (17), we get:

$$A / \delta = \frac{\omega^2 \varphi_1(l/2)}{2 \cdot \tilde{x}_{11} a \cdot \delta} \left\{ 1 \pm \sqrt{1 - 4\tilde{x}_{11} \cdot \omega^{-2} \left[\frac{\alpha}{\varphi_1(l/2)} - a\delta \int_0^l \varphi_1(x) dx \right]} \right\} \quad (21)$$

An amplitude-frequency characteristic plotted from (21) matches the graphical solution in Fig. 2. For the purposes of comparison and analysis, a resonance curve of the system is plotted there as well, leaving out the changes of belt tension relative to a phase of the vibration process. Fig. 2 shows that skeleton curves of the linear and nonlinear amplitude-frequency characteristics originate from one point which characterizes the frequency of natural system vibrations in the first mode.

4 Conclusion

Contrary to a linear amplitude-frequency characteristic, resonance frequencies are shifted into the region of higher values when the tension is increased due to the belt movement caused by vibrations. It helps to explain one fact in particular – the occurrence of resonance modes is possible in actual operational conditions if the frequencies are higher in comparison with those calculated for a linear system.

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